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Research investigations involving the basic understanding and applications of nonlinear wave motion and related studies of inverse scattering and numerical simulation have been carried out with a number of significant results obtained. During this period, four papers were published in journals and three preprints were written. Some of these contributions are outlined in this report.

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Nonlinear Problems in Fluid Dynamics and Inverse Scattering

Principle Investigator: Mark J. Ablowitz

Subtitle: Inverse Scattering and Nonlinear Waves

Program Manager: Dr. Reza Malek-Madani

Annual Progress Report

Research investigations involving the basic understanding and applications of nonlinear wave motion and related studies of inverse scattering and numerical simulation have been carried out with a number of significant results obtained. During this period, four papers were published in journals and three preprints were written. Some of these contributions are outlined in this report.

We have found solutions of a class of multidimensional nonlinear wave equations by applying the Inverse Scattering Transform (IST). A method to solve the associated multidimensional inverse scattering problems has been developed. Some of the nonlinear systems we have studied include the following physically interesting 2+1 (two space-one time) equations: the Davey-Stewartson system (a 2+1 analog of the nonlinear Schrödinger equation; an important equation in the study of deep water waves and nonlinear optics), the Kadomtsev-Petviashvili equation (a 2+1 analog of the Korteweg-deVries equation; governs weakly nonlinear long wave motion such as nonlinear waves in moderately shallow water) and the 2+1 Toda equations (the Toda equations are fundamental equations in lattice dynamics).

The solutions we have obtained correspond to the Cauchy initial value problem with decaying initial data and a subclass of boundary value problems. In one case the solution of the boundary value problem requires that a suitable radiation condition be imposed in order to obtain a unique solution. The radiation condition is the analog of the well known Sommerfeld radiation condition required in linear problems such as the Helmholtz equation. In order to obtain these solutions we analyze an associated two dimensional inverse scattering problem. The method of solution of this scattering problem leads us to a system of linear integral equations which result from a coupled DBAR, and nonlocal Riemann-Hilbert problem.

During this work we were led to study an interesting new one dimensional nonlinear equation: the Toda differential-delay equation. The analysis and associated inverse scattering problems are novel. The inverse problem requires one to solve an infinitely coupled system of Riemann-Hilbert problems.

Wave collapse has been an area of active study by us as well. It has been known for some time that multidimensional nonlinear Schrödinger equations have solutions which blow up in finite time. We have recently shown that the nonlinear Kadomtsev-Petviashvili equation exhibits wave collapse whenever the polynomial nonlinear term has a cubic or higher power.

We have been studying moderate to long time numerical simulations of coherent systems, most notably the nonlinear Schrödinger, sine-Gordon and modified Korteweg-deVries equations with periodic boundary conditions. We have observed that corresponding to initial data in certain parameter regimes, well known numerical schemes generate computational chaos even though the equations themselves are integrable and their solutions are not chaotic. In fact, we have shown that computational chaos can even be generated by tiny errors on the order of roundoff. This phenomena can be understood in terms of the underlying method of solution of the nonlinear equations; i.e. in terms of the associated scattering problem. Proximity of the solution to certain homoclinic manifolds, which are characterized by a subclass of special eigenvalues of the scattering problem, creates an unstable environment. Small perturbations in these regimes, created due to the numerical inaccuracies (truncation and roundoff), which are inherent in the numerical schemes, grow quickly and this leads to the chaotic phenomena.

Published Papers:

"On the Inverse Scattering Transform of the 2+1 Toda Equation", J. Villarroel and M.J. Ablowitz, *Physica D*, **65** (1993) 48-70.

"Numerical Chaos, Roundoff Errors and Homoclinic Manifolds", M.J. Ablowitz, C. Schober, and B.M. Herbst, *Phys. Rev. Lett.*, **71** (1993) 2683-2686.

"On the Method of Solution of the Differential-Delay Toda Equation", J. Villarroel and M.J. Ablowitz, *Phys. Lett. A*, **180** (1993) 413-418.

"Solutions to the 2+1 Toda Equation", J. Villarroel and M.J. Ablowitz, *J. Phys. A.*, **27** (1994) 931-941.

Preprints:

"Wave Collapse and Instability of Solitary Waves of a Generalized Kadomtsev-Petviashvili Equation, X.P. Wang, M.J. Ablowitz and H. Segur, June 1994, PAM[†] #146c.

"Effective Chaos in the Nonlinear Schrödinger Equation", M.J. Ablowitz and C. Schober, November 1993, PAM[†] #182.

"Numerical Stochasticity, Hamiltonian Integrators and the Nonlinear Schrödinger Equation", M.J. Ablowitz and C. Schober, December 1993, PAM[†] #184.

[†]Program in Applied Mathematics Report